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Maths — Class 12

Chapter(s): Applications of Derivatives

Maximum Marks: 39

Time Allowed: 1 Hour 10 Minutes

Section A — MCQ & Assertion-Reason (1 mark each)

Q1. A particle moves on the x-axis such that $x(t)=t^3-6t^2+9t$ (x in m, t in s). At what time(s) does the particle change its direction of motion? [1 mark]

- A) $t = 1$ s and $t = 3$ s
- B) $t = 0$ s and $t = 2$ s
- C) $t = 2$ s only
- D) $t = 1$ s only

Q2. For the function $f(x) = x + 1/x$ on the interval $[1, 4]$, the absolute maximum value is: [1 mark]

- A) 2 at $x = 1$
- B) $17/4$ at $x = 4$
- C) $5/2$ at $x = \sqrt{2}$
- D) $9/4$ at $x = 2$

Q3. A conical water tank has semi-vertical such that $r = (1/2)h$ (r and h in m). Water is poured so that $dV/dt = 6 \text{ m}^3/\text{h}$. Find dh/dt when $h = 3$ m. [1 mark]

- A) $8/(3\pi) \text{ (m)/(h)}$
- B) $4/(3\pi) \text{ (m)/(h)}$
- C) $2/(3\pi) \text{ (m)/(h)}$
- D) $8/\pi \text{ (m)/(h)}$

Q4. For $f(x)=|x|+x^2$, which statement is true? [1 mark]

- A) $x=0$ is a critical point but not a point of local extremum
- B) $x=0$ is a critical point and is a point of local minimum
- C) f is differentiable at $x=0$ and has a local maximum there
- D) f has a local maximum at $x=0$ because $|x|$ is maximum at 0

Q5. If $f'(x)=(x-2)^2(x+1)$, then the point $x=2$ is: [1 mark]

- A) a point of local maximum
- B) a point of local minimum
- C) neither local maximum nor local minimum (point of inflection type)
- D) not a critical point

Q6. Assertion (A): If f is differentiable on (a,b) and $f'(x) > 0$ for all $x \in (a,b)$, then f is strictly increasing on (a,b) .

Reason (R): By Mean Value Theorem, for any $x_1 < x_2$ in (a,b) , there exists $c \in (x_1, x_2)$ such that $(f(x_2) - f(x_1)) / (x_2 - x_1) = f'(c)$. [1 mark]

- A) Both A and R are true and R is the correct explanation of A.
- B) Both A and R are true but R is not the correct explanation of A.
- C) A is true but R is false.
- D) A is false but R is true.

Q7. Assertion (A): If f has a local maximum at an interior point c and f is differentiable at c , then $f'(c) = 0$.

Reason (R): At a local maximum, the derivative must be positive on both sides of c . [1 mark]

- A) Both A and R are true and R is the correct explanation of A.
- B) Both A and R are true but R is not the correct explanation of A.
- C) A is true but R is false.
- D) A is false but R is true.

Q8. Assertion (A): If $f'(x)$ changes sign from negative to positive at $x=c$, then $f(c)$ is a local maximum value.

Reason (R): Negative-to-positive sign change of f' indicates the function decreases then increases as x passes through c . [1 mark]

- A) Both A and R are true and R is the correct explanation of A.
- B) Both A and R are true but R is not the correct explanation of A.
- C) A is true but R is false.
- D) A is false but R is true.

Q9. Assertion (A): If a function f is differentiable on an open interval I and satisfies $f'(x) = 0$ for all $x \in I$, then f is constant on I .

Reason (R): For any $a < b$ in I , by the Mean Value Theorem there exists $c \in (a,b)$ such that $f(b) - f(a) = f'(c)(b-a)$, hence if $f'(x) = 0$ everywhere then $f(a) = f(b)$ for all a, b in I . [1 mark]

- A) Both A and R are true and R is the correct explanation of A.
- B) Both A and R are true but R is not the correct explanation of A.
- C) A is true but R is false.
- D) A is false but R is true.

Q10. Assertion (A): For a continuous function on a closed interval $[a,b]$, absolute maximum and absolute minimum values are guaranteed to exist.

Reason (R): Continuity on $[a,b]$ implies the function is differentiable on (a,b) . [1 mark]

- A) Both A and R are true and R is the correct explanation of A.
B) Both A and R are true but R is not the correct explanation of A.
C) A is true but R is false.
D) A is false but R is true.

Section B — Very Short Answer (2 marks each)

Q11. A spherical balloon is being inflated so that its volume increases at $48\pi \text{ cm}^3/\text{s}$. Find the rate of increase of its radius when the radius is 6 cm. Write the answer in exactly 2 sentences. [2 marks]

Q12. Find the intervals of increase/decrease of $f(x)=x/(x^2+1)$ on . Write the final answer as two intervals for increasing and two for decreasing, in exactly 2 sentences. [2 marks]

Q13. Using the first derivative test, find the nature of the critical point of $f(x)=x^3-3x^2+3x$ at $x=1$. Write the answer in exactly 2 sentences. [2 marks]

Section C — Short Answer (3 marks each)

Q14. A manufacturer's revenue and cost (in ₹) for producing x units are $R(x)=x(120-x)$ and $C(x)=20x+100$. Find the number of units that maximises profit and the maximum profit. Write the answer in exactly 3 numbered points. [3 marks]

Q15. Prove using calculus that among all rectangles of fixed perimeter P , the one with maximum area is a square. Write the proof in exactly 3 numbered points. [3 marks]

Q16. Determine all points of local maxima/minima (if any) for $f(x)=x^4-4x^3$ on using the first derivative test, and state the nature at each critical point. Write the answer in exactly 3 numbered points. [3 marks]

Section D — Long Answer (5 marks each)

Q17. The given figure shows an inverted right circular conical tank with vertex at V and axis vertical. Water is poured into the tank at a constant rate of $5 \text{ m}^3/\text{h}$. The semi-vertical $\angle\theta$ of the cone satisfies $\tan \theta = \frac{1}{2}$. Using the given figure/data, answer the following in one continuous solution:

- 1) Find the rate of rise of water level dh/dt when the water depth is $h = 4 \text{ m}$.
- 2) At the same instant, find the rate of change of the wetted curved surface area S of the water surface (i.e., the lateral area of the water-filled cone) with time. Show all steps and write final answers with SI units. [5 marks]

Q18. A closed rectangular box is to be made with a square base of side x cm and height h cm. The total surface area is fixed at 216 cm^2 . Using derivatives, find the dimensions x and h that maximise the volume of the box. Your solution must explicitly:

- 1) Form the constraint equation,**
- 2) Write volume as a function of one variable,**
- 3) Differentiate and find the critical point,**
- 4) Justify that it gives maximum volume,**
- 5) State the maximum volume (with unit). No part of the question has any internal choice. [5 marks]**

Section E — Case Study (4 marks each)
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Q19. Read the following passage and answer the questions that follow: [4 marks]

A drone is required to deliver a medical kit to a point near a straight road. The road is taken as the x -axis. The drone's planned vertical profile above the road is modeled by the function

$$h(x) = -x^2 + 6x + 5,$$

where x is the horizontal distance from the starting point along the road (in metres) and $h(x)$ is the height of the drone above the road (in metres). For safety, the drone must stay above a minimum height of 8 m during delivery. The drone's on-board controller adjusts speed based on the instantaneous rate of change of height with respect to x .

An engineer also monitors the "steepness" of the trajectory using the slope of the tangent to the height curve. The flight is considered smooth in an interval if $h(x)$ is increasing throughout that interval, and descending if $h(x)$ is decreasing throughout that interval. The kit is to be released exactly when the drone is at its maximum height to maximise signal strength for communication.

- (i) Find the horizontal position x (in metres) at which the drone releases the kit (maximum height point). (1 mark)
- (ii) State the interval(s) in $(0, 6)$ where the drone is ascending and where it is descending. (1 mark)
- (iii) Determine the set of x -values (in metres) for which the safety condition $h(x) \geq 8$ m is satisfied. Write the final answer as an interval (or union of intervals) with units. (2 marks)

Maths — Class 12
Chapter(s): Applications of Derivatives

Section A — MCQ & Assertion-Reason (1 mark each)

A1.

Answer:

Correct option: (A)

$t = 1 \text{ s}$ and $t = 3 \text{ s}$

Explanation:

Direction changes when velocity $v(t)=dx/dt$ changes sign at a zero. Here $v(t)=3t^2-12t+9=3(t-1)(t-3)$, which changes sign at both $t=1$ and $t=3$. Hence, option (A) is correct.

A2.

Answer:

Correct option: (B)

$17/4$ at $x = 4$

Explanation:

On $[1,4]$, $f'(x) = 1 - 1/x^2$, so the only critical point is $x = 1$ (already an endpoint). Since $f'(x) > 0$ for $x > 1$, f is increasing on $(1,4]$, hence the absolute maximum occurs at $x = 4$ with value $f(4) = 4 + 1/4 = 17/4$. Hence, option (B) is correct.

A3.

Answer:

Correct option: (A)

$8/(3\pi) \text{ [m] / [h]}$

Explanation:

Volume $V=(1/3)\pi r^2 h=(1/3)\pi(h^2/4)h=\pi h^3/12$. So $dV/dt=(\pi/4)h^2 \cdot dh/dt$; with $h=3$, $6=(\pi/4) \cdot 9 \cdot dh/dt \Rightarrow dh/dt=24/(9\pi)=8/(3\pi) \text{ [m] / [h]}$. Hence, option (A) is correct.

A4.

Answer:

Correct option: (B)

$x=0$ is a critical point and is a point of local minimum

Explanation:

At $x=0$, f is not differentiable because

$|x|$ is not differentiable at 0, so 0 is a critical point. For $x<0$, $f(x)=-x+x^2$ gives $f'(x)=-1+2x<0$ near 0; for $x>0$, $f(x)=x+x^2$ gives $f'(x)=1+2x>0$ near 0, so f decreases then increases, giving a local minimum at

0. Hence, option (B) is correct.

A5.

Answer:

Correct option: (C)

neither local maximum nor local minimum (point of inflection type)

Explanation:

$x=2$ is a critical point since $f'(2)=0$. Because $(x-2)^2 \geq 0$, the sign of $f'(x)$ near 2 is governed by $(x+1)$, which is positive on both sides of $x=2$, so f' does not change sign across 2; hence no local extremum at $x=2$. Hence, option (C) is correct.

A6.

Answer:

Correct option: (A)

Both A and R are true and R is the correct explanation of A.

Explanation:

R correctly states MVT, and with $f'(c)>0$ it implies $f(x_2)-f(x_1)>0$ for $x_2>x_1$, establishing strict increase. Hence, option (A) is correct.

A7.

Answer:

Correct option: (C)

A is true but R is false.

Explanation:

A is a necessary condition (Theorem on local extrema) when differentiable. R is false because for a local maximum, typically f' changes sign from positive (left) to negative (right), not positive on both sides. Hence, option (C) is correct.

A8.

Answer:

Correct option: (D)

A is false but R is true.

Explanation:

R is true and describes the correct pattern for a local minimum. Therefore the assertion is false because negative-to-positive change of f' gives a local minimum, not a local maximum. Hence, option (D) is correct.

A9.

Answer:

Correct option: (A)

Both A and R are true and R is the correct explanation of A.

Explanation:

Assertion (A) is true: if the derivative is identically zero on an interval, the function cannot increase or decrease anywhere on that interval. Reason (R) correctly uses Mean Value Theorem on any sub-interval $[a,b]$ to get $f(b)-f(a)=f'(c)(b-a)=0$, so $f(b)=f(a)$ for all a,b , implying f is constant; thus R is the correct explanation of A.

A10.

Answer:

Correct option: (C)

A is true but R is false.

Explanation:

A is true by the Extreme Value Theorem. R is false because continuity does not imply differentiability (e.g., $|x|$ is continuous but not differentiable at 0). Hence, option (C) is correct.

Section B — Very Short Answer (2 marks each)

A11.

Answer:

Given: $V=(4/3)\pi r^3$, so $dV/dt=4\pi r^2 \cdot dr/dt$; at $r=6$ cm and $dV/dt=48\pi$ cm³/s, $dr/dt=[48\pi] / [4\pi \cdot 36]=1/3$ cm/s. **Verification:** substituting $dr/dt=1/3$ and $r=6$ into $4\pi r^2 \cdot dr/dt$ gives $4\pi \cdot 36 \cdot (1/3)=48\pi$ cm³/s, matching the

Given: rate.

at $r=6$ cm and $dV/dt=48\pi$ cm³/s, $dr/dt=[48\pi] / [4\pi \cdot 36]=1/3$ cm/s. **Verification:** substituting $dr/dt=1/3$ and $r=6$ into $4\pi r^2 \cdot dr/dt$ gives $4\pi \cdot 36 \cdot (1/3)=48\pi$ cm³/s, matching the

Given: rate.

A12.

Answer:

Compute $f'(x) = (1 \cdot (x^2+1) - x \cdot 2x) / (x^2+1)^2 = [1-x^2] / [x^2+1]^2$, so $f'(x) > 0$ when; $|x| < 1$ and $f'(x) < 0$ when; $|x| > 1$. Hence f is increasing on $(-1, 1)$ and decreasing on $(-\infty, -1)$ and $(1, \infty)$, verified by testing $x=0$ gives $f'(0) > 0$ and $x=2$ gives $f'(2) < 0$.

$|x| > 1$. Hence f is increasing on $(-1, 1)$ and decreasing on $(-\infty, -1)$ and $(1, \infty)$, verified by testing $x=0$ gives $f'(0) > 0$ and $x=2$ gives $f'(2) < 0$.

A13.

Answer:

$f'(x) = 3x^2 - 6x + 3 = 3(x-1)^2$, so $f'(x) = 0$ at $x=1$ and $f'(x) \geq 0$ on both sides of 1 with no sign change, hence $x=1$ is neither a local maximum nor a local minimum (it is a stationary point of inflection type). Verification: taking $x=0.9$ and $x=1.1$ gives $f'(0.9) = 3(-0.1)^2 > 0$ and $f'(1.1) = 3(0.1)^2 > 0$, confirming no sign change.

Section C — Short Answer (3 marks each)
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A14.

Answer:

1) Profit function: $P(x) = R(x) - C(x) = x(120 - x) - (20x + 100) = -x^2 + 100x - 100$. Since $P(x)$ is a quadratic opening downward, its maximum occurs at the vertex where $P'(x) = 0$.

2) Critical point: $P'(x) = -2x + 100$; set $-2x + 100 = 0$ $x = 50$ units. Verification step: $P''(x) = -2 < 0$ confirms $x = 50$ gives a maximum.

3) Maximum profit: $P(50) = -(50)^2 + 100(50) - 100 = -2500 + 5000 - 100 = 2400$.

Produce 50 units for maximum profit ₹2400.

A15.

Answer:

1)

Given: & set-up: Let the rectangle have sides x and y (in m) with fixed perimeter $2(x+y) = P$ $y = (P/2) - x$. Area $A(x) = xy = x((P/2) - x) = (P/2)x - x^2$.

2) Maximisation: Differentiate $A'(x) = (P/2) - 2x$ and set $A'(x) = 0$ $x = P/4$. Verification step: $A''(x) = -2 < 0$, so $x = P/4$ gives a maximum area.

3) Conclude square: With $x = P/4$, $y = (P/2) - (P/4) = P/4$, hence $x = y$ and the rectangle is a square.

Maximum area occurs for a square of side $P/4$ m.

Hence Proved.

A16.

Answer:

1) Critical points: $f'(x) = 4x^3 - 12x^2 = 4x^2(x-3)$. Hence $f'(x) = 0$ at $x=0$ and $x=3$; these are the only

critical points.

2) Sign analysis near $x=0$: Since $4x^2 \geq 0$, the sign of $f'(x)$ near 0 is the sign of $(x-3)$, which is negative for $x < 3$; thus for x slightly left/right of 0, $f'(x) < 0$ (no sign change). Verification: $f'(-1) = 4(1)(-4) < 0$ and $f'(1) = 4(1)(-2) < 0$.

3) Sign analysis near $x=3$: For $x < 3$, $f'(x) < 0$; for $x > 3$, $f'(x) > 0$, so f' changes from - to + at $x=3$ giving a local minimum there, with $f(3) = 81 - 108 = -27$.

$x=3$ is a local minimum (value -27); $x=0$ is neither max nor min (stationary inflection-type).

Section D — Long Answer (5 marks each)

A17.

Answer:

Introduction: We relate the changing volume and surface area of the water cone using $r = h \tan \theta$ and the chain rule.

1) Geometry relation (using $\tan \theta = 1/2$):

Formula: $r = h \tan \theta = h/2$.

Step 1: When h varies with time t , r also varies with t with $r = h/2$.

Step 2: Hence $dr/dt = (1/2) dh/dt$.

2) Rate of rise of level using volume:

Formula: $V = (1/3)\pi r^2 h$.

Step 1: Substitute $r = h/2 \Rightarrow V = (1/3)\pi(h^2/4)h = (\pi/12)h^3$.

Step 2: Differentiate w.r.t. time: $dV/dt = (\pi/12) \cdot 3h^2 \cdot (dh/dt) = (\pi/4)h^2(dh/dt)$.

Step 3:

Given: $dV/dt = 5 \text{ m}^3/\text{h}$ and $h = 4 \text{ m}$; $5 = (\pi/4) \cdot (16) \cdot (dh/dt) = 4\pi(dh/dt)$; So $dh/dt = 5/(4\pi) \text{ [m] / [h]}$.

3) Wetted curved surface area S :

Formula: $S = \pi r l$, where $l = \sqrt{r^2 + h^2}$ is the slant height of water cone.

Step 1: With $r = h/2$, $l = \sqrt{(h^2/4) + h^2} = \sqrt{5h^2/4} = [h\sqrt{5}] / [2]$

Step 2: So $S = \pi \cdot (h/2) \cdot (h\sqrt{5}/2) = (\pi\sqrt{5}/4)h^2$.

4) Rate of change of S :

Step 1: Differentiate: $dS/dt = (\pi\sqrt{5}/4) \cdot 2h \cdot (dh/dt) = (\pi\sqrt{5}/2)h(dh/dt)$.

Step 2: Substitute $h = 4 \text{ m}$ and $dh/dt = 5/(4\pi) \text{ [m] / [h]}$; $dS/dt = (\pi\sqrt{5}/2) \cdot 4 \cdot (5/(4\pi)) = [5\sqrt{5}] / [2] \text{ m}^2/\text{h}$.

5)

At $h = 4$ m,; • $dh/dt = 5/(4\pi)$ [m] / [h].; • $dS/dt = [5\sqrt{5}] / [2]$ m²/h.

A18.

Answer:

Introduction: With a fixed surface area, the volume is maximised by converting the constraint to a single-variable function and;
applying derivative tests.

1) Surface-area constraint:

Formula: For a closed box with square base, $S = 2x^2 + 4xh$;

Given: $S = 216$ cm² $2x^2 + 4xh = 216$.

Step 1: Solve for h: $4xh = 216 - 2x^2$ $h = [216 - 2x^2] / [4x] = 54/x - x/2$.

2) Volume in one variable:

Formula: $V = x^2h$.

Step 1: Substitute h: $V(x) = x^2(54/x - x/2) = 54x - [x^3] / [2]$; Domain: $x > 0$ and $h > 0$
 $54/x - x/2 > 0$ $x^2 < 108$.

3) Differentiate and find critical point:

Step 1: $V'(x) = 54 - [3x^2] / [2]$

Step 2: Set $V'(x) = 0$ $54 = [3x^2] / [2]$ $x^2 = 36$ $x = 6$ cm (positive only).

Step 3: Corresponding height: $h = 54/6 - 6/2 = 9 - 3 = 6$ cm.

4) Maxima justification:

Step 1: $V''(x) = -3x$.

Step 2: At $x = 6$, $V''(6) = -18 < 0$, so V is locally maximum at $x = 6$; Also, $x = 6$ lies in $x^2 < 108$, so it is feasible.

5) Maximum volume:

Step 1: $V_{\max} = x^2h = 6^2 \times 6 = 216$ cm³.

$x = 6$ cm, $h = 6$ cm, and the maximum volume is 216 cm³.

5) Maximum volume:

Section E — Case Study (4 marks each)
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A19.

(i) Find the horizontal position x (in metres) at which the drone releases the kit (maximum height point).

Answer:

Step 1: $h(x) = -x^2 + 6x + 5$ $h'(x) = -2x +$

6.

Step 2: Maximum height occurs where $h'(x) = 0$ $-2x + 6 = 0$ $x = 3$ m.

$x = 3$ m.

(ii) State the interval(s) in (0, 6) where the drone is ascending and where it is descending.

Answer:

Step 1: $h'(x) = -2x +$

6.

Step 2: $h'(x) > 0$ when $x < 3$ and $h'(x) < 0$ when $x >$

3.

Ascending on (0,

3) and descending on (3, 6).

(iii) Determine the set of x-values (in metres) for which the safety condition $h(x) \geq 8$ m is satisfied. Write the final answer as an interval (or union of intervals) with units.

Answer:

Step 1: Solve $h(x) \geq 8$; $-x^2 + 6x + 5 \geq 8 \Rightarrow -x^2 + 6x - 3 \geq$

0.

Step 2: Multiply by -1 (reverse inequality): $x^2 - 6x + 3 \leq$

0.

Step 3: Roots: $x = (6 \pm \sqrt{(36 - 12)})/2 = [6 \pm \sqrt{24}] / [2] = 3 \pm \sqrt{6}$.

Step 4: Since the parabola $x^2 - 6x + 3$ opens upward, it is ≤ 0 between roots.

Safety holds for $x \in [3 - \sqrt{6}, 3 + \sqrt{6}]$ m.