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Maths — Class 11

Chapter(s): TRIGONOMETRIC FUNCTIONS

Maximum Marks: 39

Time Allowed: 1 Hour 10 Minutes

Section A — MCQ & Assertion-Reason (1 mark each)

Q1. A wheel of radius 70 cm rolls without slipping. Through what $\angle$ (in radians) does it rotate to cover a linear distance of 11 m? (Use $\pi = 22/7$.) [1 mark]

- A) $22/7$
- B) $110/7$
- C) $110/49$
- D) $11/70$

Q2. If $\cos x = [-12] / [13]$ and x lies in the second quadrant, then the value of $\sin(x/2)$ is: [1 mark]

- A) $5/\sqrt{26}$
- B) $\sqrt{(25/26)}$
- C) $\sqrt{(1/26)}$
- D) $1/\sqrt{26}$

Q3. The value of $\tan(13\pi/12)$ is: [1 mark]

- A) $\sqrt{3} - 2$
- B) $2 - \sqrt{3}$
- C) $2 + \sqrt{3}$
- D) $\sqrt{3} + 2$

Q4. If $\sin x + \cos x = \sqrt{2}$, then the set of all real values of x is: [1 mark]

- A) $2n\pi + \pi/4 : n \in \mathbb{Z}$
- B) $n\pi + \pi/4 : n \in \mathbb{Z}$
- C) $2n\pi - \pi/4 : n \in \mathbb{Z}$
- D) $n\pi - \pi/4 : n \in \mathbb{Z}$

Q5. Assertion (A): For all real x , $\cos(x + y) = \cos x \cos y - \sin x \sin y$.

Reason (R): Points $(\cos t, \sin t)$ lie on the unit circle and distances between points can be compared to derive addition formulas. [1 mark]

- A) Both A and R are true and R is the correct explanation of A.
- B) Both A and R are true but R is not the correct explanation of A.
- C) A is true but R is false.
- D) A is false but R is true.

Q6. Assertion (A): $\tan(x + y) = [\tan x + \tan y] / [1 - \tan x \tan y]$ is valid whenever none of $x, y, x + y$ is an odd multiple of $\pi/2$.

Reason (R): The formula is obtained by dividing $\sin(x + y)$ and $\cos(x + y)$ by $\cos x \cos y$, which requires $\cos x \neq 0$ and $\cos y \neq 0$ and also $\cos(x + y) \neq 0$. [1 mark]

- A) Both A and R are true and R is the correct explanation of A.
- B) Both A and R are true but R is not the correct explanation of A.
- C) A is true but R is false.
- D) A is false but R is true.

Q7. Assertion (A): For all real x , the range of $y = \tan x$ is the set of all real numbers.

Reason (R): In each interval $((2n-1)\pi/2, (2n+1)\pi/2)$, $n \in \mathbb{Z}$, the function $\tan x$ is continuous and increases from $-\infty$ to $+\infty$. [1 mark]

- A) Both A and R are true and R is the correct explanation of A.
- B) Both A and R are true but R is not the correct explanation of A.
- C) A is true but R is false.
- D) A is false but R is true.

Q8. Assertion (A): The identity $\cos(2x) = [1 - \tan^2 x] / [1 + \tan^2 x]$ is valid for every real x .

Reason (R): Dividing the identity $\cos(2x) = \cos^2 x - \sin^2 x$ by $\cos^2 x + \sin^2 x$ gives $\cos(2x) = [\cos^2 x - \sin^2 x] / [\cos^2 x + \sin^2 x]$, and then dividing numerator and denominator by $\cos^2 x$ yields $[1 - \tan^2 x] / [1 + \tan^2 x]$. [1 mark]

- A) Both A and R are true and R is the correct explanation of A.
- B) Both A and R are true but R is not the correct explanation of A.
- C) A is true but R is false.
- D) A is false but R is true.

Q9. Assertion (A): $\tan x$ has period 2π .

Reason (R): $\sin x$ and $\cos x$ both have period 2π , and $\tan x = \sin x / \cos x$. [1 mark]

- A) Both A and R are true and R is the correct explanation of A.
- B) Both A and R are true but R is not the correct explanation of A.
- C) A is true but R is false.
- D) A is false but R is true.

Section B — Very Short Answer (2 marks each)

Q10. Convert $343^\circ 38' 11''$ into radians. Give the answer correct to the nearest 0.001 rad. (Use $\pi \approx 22/7$.) [2 marks]

Q11. If $\cot x = [-5] / [12]$ and x lies in the second quadrant, find $\sec x$ and $\operatorname{cosec} x$. (Write the final values as rational numbers.) [2 marks]

Q12. Solve for all real x : $\sin x = \cos x$. Write the general solution in radians. [2 marks]

Section C — Short Answer (3 marks each)
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Q13. Prove that $\tan 3x \cdot \tan 2x \cdot \tan x = \tan 3x - \tan 2x - \tan x$. (Answer in exactly 3 numbered points, each with a clear heading.) [3 marks]

Q14. Using only sum-to-product identities, simplify $[\cos 7x + \cos 5x] / [\sin 7x - \sin 5x]$ to a single trigonometric function. (Answer in exactly 3 numbered points.) [3 marks]

Q15. A rotating laser makes two sweeps in the same time interval. In sweep-1 it turns through 65° , and in sweep-2 through 110° . If the arc lengths traced on two circular screens are equal, find the ratio of the radii $r_1 : r_2$. (Answer in exactly 3 numbered points.) [3 marks]

Section D — Long Answer (5 marks each)

Q16. The given figure shows a lighthouse AB of height 60 m standing vertically on level ground at B. Two ships are at points P and Q on the same straight line from B on the sea surface, on the same side of the lighthouse. The $\angle$ of elevation of the top A from P is 30° and from Q is 60° . (i) Find BP and BQ (in m). (ii) Find the distance PQ (in m). (iii) Find the $\angle$ subtended by PQ at A, i.e., $\angle PAQ$ (in degrees). Use exact values of standard trigonometric ratios. [5 marks]

Q17. Prove the identity (show all steps clearly): $\sin 3\theta = 3\sin \theta - 4\sin^3\theta$. Then, using this identity, find all values of θ in $[0, 2\pi]$ for which $\sin 3\theta = \sin \theta$. [5 marks]

Section E — Case Study (4 marks each)
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Q18. Read the following passage and answer the questions that follow: [4 marks]

A robotics lab programs a camera gimbal to rotate about a fixed point O. The rotation $\angle$ is controlled in radians, but the engineer sometimes records angles in degrees. During a calibration run, the gimbal starts aligned with the positive x-axis and rotates anticlockwise. The lab notes that after the first command, the gimbal rotates through 150° ; after the second command, it rotates back (clockwise) through $7\pi/6$ radians; after the third command, it rotates anticlockwise through $5\pi/3$ radians. The gimbal's orientation determines the unit-circle point $P(\cos \theta, \sin \theta)$ where θ is the final net rotation from the positive x-axis. To avoid collisions, the software must also confirm whether $\sin \theta$ and $\cos \theta$ are positive/negative and compute the exact value of $\cos(\theta + \pi/6)$ for the final θ .

(i) Find the final net rotation $\angle \theta$ in radians, reduced to an equivalent $\angle$ in the interval $[0, 2\pi)$. (1 mark)

- (ii) State the signs of $\sin \theta$ and $\cos \theta$ for this final $\angle \theta$. (1 mark)
- (iii) Compute the exact value of $\cos(\theta + \pi/6)$ for the final θ . (2 marks)

Maths — Class 11
Chapter(s): TRIGONOMETRIC FUNCTIONS

Section A — MCQ & Assertion-Reason (1 mark each)

A1.

Answer:

Correct option: (B)

$$110/7$$

Explanation:

For rolling without slipping, arc length $l = r\theta$. Here $l = 11 \text{ m} = 1100 \text{ cm}$ and $r = 70 \text{ cm}$, so $\theta = [l] / [r] = 1100/70 = 110/7 \text{ rad}$. Hence, option (B) is correct.

A2.

Answer:

Correct option: (B)

$$\sqrt{(25/26)}$$

Explanation:

Since x is in QII, $x/2$ lies in QI, so $\sin(x/2) > 0$. Using $\sin^2(x/2) = [1 - \cos x] / [2] = (1 - ([-12] / [13])) / 2 = [25/13] / [2] = 25/26$, hence $\sin(x/2) = \sqrt{(25/26)}$. Hence, option (B) is correct.

A3.

Answer:

Correct option: (B)

$$2 - \sqrt{3}$$

Explanation:

$\tan(13\pi/12) = \tan(\pi + \pi/12) = \tan(\pi/12)$. Now $\tan(\pi/12) = \tan(\pi/4 - \pi/6) = [1 - 1/\sqrt{3}] / [1 + 1/\sqrt{3}] = 2 - \sqrt{3}$. Hence, option (B) is correct.

A4.

Answer:

Correct option: (A)

$$2n\pi + \pi/4 : n \in \mathbb{Z}$$

Explanation:

$\sin x + \cos x = \sqrt{2} \sin(x+\pi/4)$. So $\sqrt{2} \sin(x+\pi/4) = \sqrt{2} \Rightarrow \sin(x+\pi/4) = 1 \Rightarrow x+\pi/4 = \pi/2 + 2n\pi \Rightarrow x = \pi/4 + 2n\pi$. Hence, option (A) is correct.

A5.

Answer:

Correct option: (A)

Both A and R are true and R is the correct explanation of A.

Explanation:

The cosine addition identity is true for all real x, y . The reason states the unit-circle coordinate method used to derive the identity by comparing lengths/distances between points corresponding to angles. Hence, option (A) is correct.

A6.

Answer:

Correct option: (A)

Both A and R are true and R is the correct explanation of A.

Explanation:

The identity for $\tan(x+y)$ comes from $\tan(x+y) = \sin[x+y] / [\cos](x+y)$ and the sum

Formulas: , and the exclusions ensure denominators are non-zero. The reason correctly cites the division step and the necessity of $\cos x$, $\cos y$, and $\cos(x+y)$ being non-zero. Hence, option (A) is correct.

A7.

Answer:

Correct option: (A)

Both A and R are true and R is the correct explanation of A.

Explanation:

The assertion is true because $\tan x$ attains every real value. The reason correctly explains this: between consecutive vertical asymptotes at odd multiples of $\pi/2$, $\tan x$ is continuous and strictly increasing, taking values from $-\infty$ to $+\infty$, which forces the overall range to be $\mathbb{R}$. Hence, option (A) is correct.

A8.

Answer:

Correct option: (D)

A is false but R is true.

Explanation:

The assertion is false because the expression $[1 - \tan^2 x] / [1 + \tan^2 x]$ requires $\tan x$ to be defined, i.e., $\cos x \neq 0$, so it is not valid for every real x . The reason is true as it gives a correct derivation on the domain where $\cos x \neq 0$ (so division by $\cos^2 x$ is permitted). Hence, option (D) is correct.

A9.

Answer:

Correct option: (D)

A is false but R is true.

Explanation:

$\tan x$ actually repeats after π , so the assertion is false. The reason statement is true (sin and cos have period 2π and $\tan x$ is their ratio), but it does not imply the minimal period is 2π . Hence, option (D) is correct.

Section B — Very Short Answer (2 marks each)

A10.

Answer:

Formula: radians = $(\pi/180) \times$ degrees.

Step 1: $343^\circ 38' 11'' = 343 + 38/60 + 11/3600 = 343.6364^\circ$ (approx).

Step 2: radians $\approx [22/7] / [180] \times 343.6364 = [22 \times 343.6364] / [1260] \approx 6.000$ rad.
6.000 rad.

A11.

Answer:

Formula: $1 + \cot^2 x = \operatorname{cosec}^2 x$ and $\sec x = 1/\cos x$.

Step 1: $\cot x = [-5] / [12]$ $\tan x = [-12] / [5]$.

Step 2: $\sec^2 x = 1 + \tan^2 x = 1 + (144/25) = 169/25$ $\sec x = [-13] / [5]$ (negative in QII).

Step 3: $\cos x = 1/\sec x = [-5] / [13]$ and $\sin x = \tan x \cdot \cos x = ([-12] / [5])([-5] / [13]) = 12/13$ $\operatorname{cosec} x = 13/12$.

$\sec x = [-13] / [5]$, $\operatorname{cosec} x = 13/12$.

A12.

Answer:

Formula: $\sin x = \cos x \Rightarrow \tan x = 1$ (when $\cos x \neq 0$).

Step 1: $\sin x = \cos x$ $\sin x / \cos x = 1$ $\tan x =$
1.

Step 2: $\tan x = 1 \Rightarrow x = \pi/4 + n\pi, n \in \mathbb{Z}$.

$$x = \pi/4 + n\pi, n \in \mathbb{Z}.$$

Section C — Short Answer (3 marks each)
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A13.

Answer:

1) Heading: Start from $\tan(2x + x)$; We use $3x = 2x + x$ and the identity $\tan(A + B) = [\tan A + \tan B] / [1 - \tan A \tan B]$ (valid when denominators are non-zero).

2) Heading: Rearrangement; $\tan 3x = [\tan 2x + \tan x] / [1 - \tan 2x \tan x]$ $\tan 3x(1 - \tan 2x \tan x) = \tan 2x + \tan x$.

3) Heading: Final factor form; $\tan 3x - \tan 3x \cdot \tan 2x \cdot \tan x = \tan 2x + \tan x$ $\tan 3x \cdot \tan 2x \cdot \tan x = \tan 3x - \tan 2x - \tan x$.

$$\tan 3x \cdot \tan 2x \cdot \tan x = \tan 3x - \tan 2x - \tan x.$$

Hence Proved.

A14.

Answer:

1) Heading: Apply $\cos A + \cos B$; $\cos 7x + \cos 5x = 2\cos([7x+5x] / [2])\cos([7x-5x] / [2]) = 2\cos 6x \cdot \cos x$.

2) Heading: Apply $\sin A - \sin B$; $\sin 7x - \sin 5x = 2\cos([7x+5x] / [2])\sin([7x-5x] / [2]) = 2\cos 6x \cdot \sin x$.

3) Heading: Cancel common factor and conclude; $[\cos 7x + \cos 5x] / [\sin 7x - \sin 5x] = [2\cos 6x \cos x] / [2\cos 6x \sin x] = \cos x / \sin x = \cot x$.

$$\cot x.$$

$$[\cos 7x + \cos 5x] / [\sin 7x - \sin 5x] = [2\cos 6x \cos x] / [2\cos 6x \sin x] = \cos x / \sin x = \cot x.$$

A15.

Answer:

1) Heading: Use arc-length relation; For a circle, arc length $l = r\theta$ where θ must be in radians.

2) Heading: Convert angles and equate arc lengths; $\theta_1 = 65^\circ = (65\pi/180) = 13\pi/36$, and $\theta_2 = 110^\circ = (110\pi/180) = 22\pi/36$; Equal arc lengths give $r_1\theta_1 = r_2\theta_2 \Rightarrow r_1(13\pi/36) = r_2(22\pi/36)$.

3) Heading: Solve for ratio; Cancel $\pi/36$ to get $13r_1 = 22r_2 \Rightarrow r_1/r_2 = 22/13$.

$$r_1 : r_2 = 22 :$$

13.

$$\text{Cancel } \pi/36 \text{ to get } 13r_1 = 22r_2 \Rightarrow r_1/r_2 = 22/13.$$

Section D — Long Answer (5 marks each)

A16.

Answer:

Introduction: We use right-triangle trigonometry and the identities for tan, along with the cosine rule (or dot-product idea) to obtain distances and the at the top.

1) Interpreting the figure (Right triangles); In $\triangle ABP$, $AB \perp BP$, $\angle$ of elevation at P is $\angle APB = 30^\circ$. In $\triangle ABQ$, $AB \perp BQ$ and $\angle AQB = 60^\circ$; Thus, both triangles are right-angled at B and share the common height $AB = 60$ m.

2) Finding BP using $\tan 30^\circ$:

Formula: $\tan \theta = [\text{opposite}] / [\text{adjacent}] = AB/BP$; So $\tan 30^\circ = 60/BP \Rightarrow 1/\sqrt{3} = 60/BP \Rightarrow BP = 60\sqrt{3}$ m.

3) Finding BQ using $\tan 60^\circ$:

Formula: $\tan \theta = AB/BQ$; So $\tan 60^\circ = 60/BQ \Rightarrow \sqrt{3} = 60/BQ \Rightarrow BQ = 60/\sqrt{3} = 20\sqrt{3}$ m.

4) Finding PQ on the same line; Since P and Q lie on the same ray from B, $PQ = BP - BQ$; Therefore $PQ = 60\sqrt{3} - 20\sqrt{3} = 40\sqrt{3}$ m.

5) Finding $\angle PAQ$ ($\angle$ subtended at the top); In $\triangle APQ$, compute AP and AQ first.; $AP = AB/\sin 30^\circ = 60/(1/2) = 120$ m; $AQ = AB/\sin 60^\circ = 60/(\sqrt{3}/2) = 40\sqrt{3}$ m.; Now $PQ = 40\sqrt{3}$ m. Using cosine rule at A; $\cos \angle PAQ = [AP^2 + AQ^2 - PQ^2] / [2 \cdot AP \cdot AQ] = (120^2 + (40\sqrt{3})^2 - (40\sqrt{3})^2) / (2 \cdot 120 \cdot 40\sqrt{3}) = 14400 / (9600\sqrt{3}) = \sqrt{3}/2$; Hence $\angle PAQ = 30^\circ$.

Hence $\angle PAQ = 30^\circ$.

A17.

Answer:

Introduction: We first derive $\sin 3\theta$ using the sum

Formula: $\sin(\alpha+\beta)$, and then solve the resulting trigonometric equation on $[0, 2\pi]$.

1) Start with the $\angle$ addition setup; Use $3\theta = 2\theta + \theta$ and the identity $\sin(\alpha+\beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$; So, $\sin 3\theta = \sin(2\theta+\theta) = \sin 2\theta \cos \theta + \cos 2\theta \sin \theta$.

2) Substitute double- $\angle$ expressions; We know $\sin 2\theta = 2\sin \theta \cos \theta$ and $\cos 2\theta = 1 - 2\sin^2\theta$; Hence $\sin 3\theta = (2\sin \theta \cos \theta)\cos \theta + (1 - 2\sin^2\theta)\sin \theta$.

3) Simplify to a polynomial in $\sin \theta$; $\sin 3\theta = 2\sin \theta \cos^2\theta + \sin \theta - 2\sin^3\theta$; Using $\cos^2\theta = 1 - \sin^2\theta$, we get $2\sin \theta(1 - \sin^2\theta) + \sin \theta - 2\sin^3\theta$.

4) Collect like terms; $= 2\sin \theta - 2\sin^3\theta + \sin \theta - 2\sin^3\theta = 3\sin \theta - 4\sin^3\theta$; Thus the identity is proved.

5) Solve $\sin 3\theta = \sin \theta$ on $[0, 2\pi]$; Using the identity: $3\sin \theta - 4\sin^3\theta = \sin \theta$; So $2\sin \theta - 4\sin^3\theta = 0 \Rightarrow 2\sin \theta(1 - 2\sin^2\theta) = 0$; Case A: $\sin \theta = 0 \Rightarrow \theta = 0, \pi, 2\pi$; Case B: $1 - 2\sin^2\theta$

$$= 0 \Rightarrow \sin^2 \theta = \frac{1}{2} \Rightarrow \sin \theta = \pm 1/\sqrt{2}; \text{ So } \theta = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4.$$

Hence Proved.

Conclusion: $\sin 3\theta = 3\sin \theta - 4\sin^3 \theta$, and the solutions are $\theta = 0, \pi/4, 3\pi/4, \pi, 5\pi/4, 7\pi/4, 2\pi$.

Section E — Case Study (4 marks each)
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A18.

(i) Find the final net rotation $\angle \theta$ in radians, reduced to an equivalent $\angle$ in the interval $[0, 2\pi)$.

Answer:

Step 1: Convert 150° to radians: $150^\circ = 150 \times (\pi/180) = 5\pi/6$.

Step 2: Signed sum (anticlockwise positive): $\theta = 5\pi/6 - 7\pi/6 + 5\pi/3$.

Step 3: Compute: $5\pi/6 - 7\pi/6 = -2\pi/6 = -\pi/3$; then $-\pi/3 + 5\pi/3 = 4\pi/3$.
 $\theta = 4\pi/3$ rad.

(ii) State the signs of $\sin \theta$ and $\cos \theta$ for this final $\angle \theta$.

Answer:

Step 1: $\theta = 4\pi/3$ lies in the third quadrant ($\pi < \theta < 3\pi/2$).

Step 2: In quadrant III, both x- and y-coordinates on the unit circle are negative.
 $\sin \theta < 0$ and $\cos \theta < 0$.

(iii) Compute the exact value of $\cos(\theta + \pi/6)$ for the final θ .

Answer:

Step 1: $\theta + \pi/6 = 4\pi/3 + \pi/6 = 8\pi/6 + \pi/6 = 9\pi/6 = 3\pi/2$.

Step 2: $\cos(3\pi/2) =$

0.

$\cos(\theta + \pi/6) = 0$.